Question:

Which of the following parametrizes the straight line from $(1, 3)$ to $(0, 1)$?

a. $(0,1)\,t + (1,3)\,(1-t)$

When $t = 0$ we get $(1,3)$
when $t = 1$ we get $(0,1)$

b. $(1,3) - (1,2)\,t$

c. $(0,1) + (1,2)\,(1-t)$

d. All of the above.

e. None of the above.

Section 4.2: Arc length

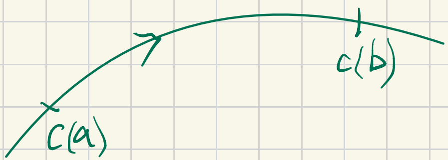
We learn:

- How to compute the length of a curve as an integral.
- A couple of ways of writing it. You need to be able to write it down either from memory or from understanding how the integral is constructed.

All the HW questions ask you to compute the length of a parametrized curve. Some of the questions are dressed up a bit.

Approach 1 to the length of a curve:

The curve is parametrized by $c(t)$ with $a \le t \le b$.



Now at any instant the length increases by the speed $\|c'(t)\|$ times the time increase.
The length is:

$$\int_a^b \|c'(t)\|\, dt$$

Example. Let $c(t) = (\cos(t), \sin(t), t)$. Find the arc length along the curve between $t = 0$ and $t = 2\pi$.

Solution: $c'(t) = (-\sin t, \cos t, 1)$

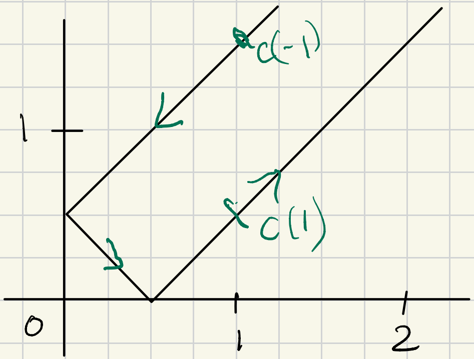
The length is

$$\int_0^{2\pi} \sqrt{(-\sin t)^2 + \cos^2 t + 1^2}\, dt$$

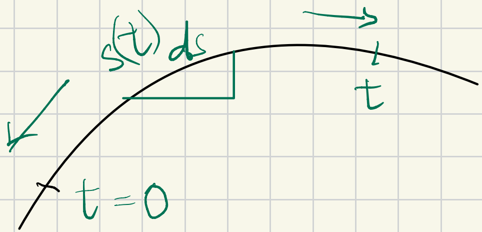
$$= \int_0^{2\pi} \sqrt{1 + 1}\, dt = \int_0^{2\pi} \sqrt{2}\, dt$$

$$= 2\sqrt{2}\pi.$$

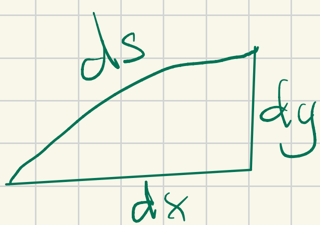
Example 3 in the book:
c: [-1,1] -> R^2 is defined by c(t) = (|t|, |t- 1/2|).
Find the length.



Second approach to theory.



s(t) = length from t = 0 to current value of t.



$$ds^2 = dx^2 + dy^2.$$
$$ds = \sqrt{dx^2 + dy^2}$$

Length

$$\int_a^b ds = \int_a^b \sqrt{dx^2 + dy^2}$$

If $(x, y) = c(t)$ we write this as

$$\int_a^b \sqrt{dx^2 + dy^2} = \int_a^b \sqrt{\frac{dx^2}{dt^2} + \frac{dy^2}{dt^2}}\, dt$$

$$= \int_a^b \| c'(t) \| \, dt$$